GROUPS IN WHICH THE NORMALISER OF EVERY ELEMENT EXCEPT IDENTITY IS ABELIAN*

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1. *Introduction.* Certain groups, of which the tetrahedral and icosahedral groups are non-trivial examples, possess the property that the normaliser of every element except identity is abelian. In this paper we shall investigate the properties of such groups.

2. *Abelian Subgroups.* An abelian subgroup of a group will be called a *maximal* abelian subgroup if it is not contained in any larger abelian subgroup of the group.

If two abelian subgroups of a group G, in which the normaliser of every element except identity is abelian, have an element besides identity in common, they generate an abelian group; for an element common to the abelian subgroups H and I is invariant under (H, I), which must therefore be abelian. It follows that *the maximal abelian subgroups of G are independent*; that is, no two maximal abelian subgroups of G have an element in common besides identity.

Since every prime-power group possesses an invariant element besides identity, *the Sylow subgroups of G are abelian.* Moreover, *the Sylow subgroups of G are independent.* For if two Sylow subgroups H and I (necessarily of the same order p^a) had an element in common besides identity, (H, I) would be an abelian subgroup of G of order $p^b > p^a$, which contradicts the fact that p^a is the highest power of p that divides the order of G.

Let H be a maximal abelian subgroup of G, I a Sylow subgroup of H, and J that Sylow subgroup of G which includes I. Since H and J have I in common, (H, J) is

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