ON SETS OF THREE CONSECUTIVE INTEGERS WHICH ARE QUADRATIC RESIDUES OF PRIMES*

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In this paper we shall prove the following theorems.

THEOREM I. *For each prime, p, for which there are as many as three incongruent squares, there is a set of three consecutive residues (admitting zero and negative numbers as residues) which are squares, modulo p.*

THEOREM II. *For $p = 11$, and for each prime p greater than 17, (and for no other primes), there is a set of three consecutive least positive (non-zero) residues which are squares, modulo p.*

The problem† of finding three consecutive integers which are quadratic residues of a prime, p, is equivalent to the formally more general problem of finding two quantities, x, y, $(y \not\equiv 0)$, such that x, y, $x+y$, $x-y$, are proportional to squares in the domain,‡ since we then have $(x/y)-1$, x/y, $(x/y)+1$ as consecutive squares in the domain. We may show that for residues with respect to a modulus the condition is equivalent to the existence of a square of the form§ $u\,v\,(u+v)\,(u-v)$. By taking $u=x$, $v=y$, we see that the condition is necessary.

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† For references, compare article of similar title by H. S. Vandiver, this BULLETIN, vol. 31 (1925), p. 33.

‡ That, in the system of natural numbers, it is impossible to have distinct quantities, x, y, such that x, y, $x+y$, $x-y$ are all proportional to squares was proved by Fermat by his celebrated method of "infinite descent". See Carmichael, *Theory of Numbers*, p. 86.

§ It is of interest to note that in the case of natural numbers we may take $u=x$ and $v=y$ for this relation. Indeed, if $x, y, x+y$, $x-y$ were proportional to squares, certainly their product would be a square. Conversely, suppose that their product were a square. Then either x, y, $x+y$, $x-y$ would all be relatively prime, or if