CONCERNING THE COMPLEMENTARY INTERVALS OF COUNTABLE CLOSED SETS*

BY J. R. KLINE

Professor R. L. Moore† recently gave an example showing that the following statement due to Hobson‡ is incorrect: "*A non-dense closed set is enumerable if its complementary intervals are such that every one of them abuts on another one at each of its ends.*"

In considering this example, we notice that while each of the complementary intervals of the original set of Moore's example are such that each one of them abuts on another one at each of its ends, the same is not true if we consider the complementary intervals of G', the first derived set§ of G. This leads us to the following theorem:

THEOREM A. *A non-dense closed set G is enumerable, if G and every derived set of G has the property that every one of its complementary intervals is such that it abuts on another one at each of its ends.*

PROOF: Let us suppose that G is not enumerable. Then there exists a number β of the first or second class such that the derived set $G^{\beta+1}$ is identical with the derived set G^{β} and G^{β} is perfect.|| It follows that G^{β} has no

* Presented to the Society, October 25, 1924.

† See R. L. Moore, *An uncountable, closed, and non-dense point set, each of whose complementary intervals abuts on another one at each of its ends*, this BULLETIN, vol. 29, (1923), pp. 49–50.

‡ See E. W. Hobson, *The Theory of Functions of a Real Variable* 1st edition (1907), p. 92; and 2d edition (1921), p. 113.

§ If G is a point set, then the set G' consisting of all limit points of G is called the *derived set* or the derivative of G.

|| See E. W. Hobson, loc. cit., p. 115. In his proof of the theorem that every non-dense linear closed set is, in general, made up of an enumerable set and a perfect set, Hobson here shows that unless the non-dense linear closed set G is enumerable, there is a number β of the first or second class such that G^{β} is perfect.