NOTE ON THE PROJECTIVE GEOMETRY OF PATHS

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1. *Projective Geometry of Paths.* It was first shown by Weyl† that the functions $\Gamma^i_{\alpha\beta}$ and the functions

$$(1) \qquad \Lambda^i_{\alpha\beta} = \Gamma^i_{\alpha\beta} + \delta^i_\alpha \psi_\beta + \delta^i_\beta \psi_\alpha,$$

where ψ_α is an arbitrary covariant vector, and

$$\delta^i_\alpha = 0, \quad \text{for } i \neq \alpha; \qquad = 1, \quad \text{for } i = \alpha,$$

define the same geometry of paths. This leads to the consideration of properties of the paths which are independent of the particular set of functions $\Gamma^i_{\alpha\beta}$ by means of which the paths are defined. Theorems expressing such properties constitute the projective geometry of paths. In the following note we give a few theorems belonging to the projective geometry of paths.

2. *Projective Tensors.* Theorems of the projective geometry of paths appear to have their statement in terms of what may be called *projective tensors,* i. e. tensors which are independent of the particular set of functions $\Gamma^i_{\alpha\beta}$ defining the paths. We shall show how a set of projective tensors may be derived by covariant differentiation from an n-uple of mutually independent vectors.

Let $h_{(\alpha)i}$ denote an n-uple of independent covariant vectors. Then the determinant

$$(2) \qquad h = |h_{(\alpha)i}|$$

does not vanish identically. We may therefore define an n-uple of contravariant vectors $h^{(\alpha)i}$ as the cofactors of the

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† H. Weyl, Göttinger Nachrichten, 1921, p. 99. See also O. Veblen, Proceedings of the National Academy, vol. 8 (1922), p. 347; and O. Veblen and T. Y. Thomas, Transactions of this Society, vol. 25, p. 557.