ON THE NUMBER OF REPRESENTATIONS OF AN INTEGER AS A SUM OR DIFFERENCE OF TWO CUBES*

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1. *Introduction and Summary.* Let $C(n)$ denote the number of integer solutions $(x, y) = (\xi, \eta)$, $\xi > 0$, $\eta > 0$, of $x^3 + y^3 = n$, the pair (ξ, η), (η, ξ) being considered as a single solution, and $D(n)$ the number of integer solutions (x, y), $x > 0$, $y > 0$ of $x^3 - y^3 = n$, $n > 0$. If the pair (ξ, η), (η, ξ) in $C(n)$ be counted as two solutions, the total number is evidently $2C(n)$ or $2C(n) - 1$ according as n is not or is the double of an integer cube > 0. No determination of $C(n)$, $D(n)$ seems to have been made. It will be of interest therefore to record forms of these functions *depending only upon the real divisors of* n, in analogy to the classical results for $x^2 \pm y^2 = n$. These forms also indicate fairly expeditious means for finding all the resolutions of n into a sum or difference of two cubes.

We denote by $\psi(z)$ the well known function whose value is 1 or 0 according as z is or is not an integer square $\geqq 0$. In the sequel only integer arguments z occur. For $S(n) \equiv C(n)$ or $D(n)$ we find the following:

(1) $\quad n \equiv 0 \bmod 3, \qquad n \not\equiv 0 \bmod 9, \qquad S(n) = 0.$

(2) $\quad n \equiv 0 \bmod 9, \qquad S(n) = \sum \psi(4t - 3d^2),$

the $\sum$ extending to all pairs (t, d) of conjugate divisors of $n/9$ such that

$$\frac{1}{3}\sqrt[3]{n} < d \leqq \frac{1}{3}\sqrt[3]{4n}, \text{ or } d < \frac{1}{3}\sqrt[3]{n},$$

according as $S = C$ or D.

(3) $\quad n \equiv \pm 1 \bmod 3, \qquad S(n) = \sum \psi\left(\frac{4t - 3d^2}{3}\right),$

* Presented to the Society, San Francisco Section, June 19, 1925.