

we see that any term which does not correspond to an absolute permutation of the  $n$  indices  $i, j, \dots, k$  contains an element from the main diagonal and is therefore zero. The value of a term corresponding to an absolute permutation is  $+1$  or  $-1$  according as the permutation is even or odd. The value of  $D$  is therefore the difference between the number of even absolute permutations,  $N_e$ , and the number of odd absolute permutations,  $N_o$ , that is

$$(1) \quad N_e - N_o = (-1)^{n-1}(n-1).$$

Moreover we have\*

$$(2) \quad N_e + N_o = n! \sum_{r=2}^n \frac{(-1)^r}{r!}.$$

From (1) and (2)  $N_e$  and  $N_o$  can be calculated.

PRINCETON UNIVERSITY

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## THE ABSOLUTE VALUE OF THE PRODUCT OF TWO MATRICES†

BY J. H. M. WEDDERBURN

1. *Introduction.* If  $a = (a_{pq})$  is a matrix of order  $n$  whose elements are ordinary complex numbers, the absolute value of  $a$  is defined as  $\sqrt{\sum a_{pq} \bar{a}_{pq}}$ , where  $\bar{a} = (\bar{a}_{pq})$  is the matrix whose coefficients are the conjugates of the corresponding coefficients in  $a$ ; we shall denote it here by  $|a|$ , a special symbol being convenient since the absolute value of a scalar matrix  $\lambda$  is not  $|\lambda| \equiv \text{mod } \lambda$  but  $n^{1/2}|\lambda|$ . This definition has been freely used by writers on differential equations; but, in spite of this, its properties with regard to multiplication have seemingly escaped notice, or are at least not well known.

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\* Cf. Seelhoff, ARCHIV DER MATHEMATIK UND PHYSIK, (2), vol. 1, p. 100.

† Presented to the Society, May 2, 1925.