INTEGRO-DIFFERENTIAL EQUATIONS*

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1. *Introduction.* It is proposed in this note to prove that if, in the integro-differential equation

$$\frac{\partial\varphi(x,t)}{\partial t}=\int_0^1 \varkappa(x,y)\varphi(y,t)dy, \tag{1}$$

the real and continuous kernel $\varkappa(x,y)$ is also symmetric or skew-symmetric then there can be no solutions of the form

$$e^{\lambda t}\sum_{k=0}^{p}\alpha_k(x)t^k \tag{2}$$

unless all $\alpha_k(x)$, $k\geqq 1$ are identically zero.† It will then be pointed out that a similar property holds for systems of linear differential equations

$$\frac{d\varphi_i}{dt}=\sum_{j=1}^{n}\varkappa_{ij}\varphi_j, \tag{3}$$

viz., if the $\varkappa_{ij}$ are real and constant (with respect to t), then in case the coefficient system is either symmetric or skew-symmetric, no real set of solutions can contain polynomials in t as factors. This fact seems, to the best of the writer's knowledge, to have escaped attention in the literature.‡

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† See I. A. Barnett, *Integro-differential equations with constant limits of integration*, this BULLETIN, vol. 26, pp. 193–203.

‡ I have recently had the privilege of examining, through the courtesy of Professor Brand of the University of Cincinnati, a course of lectures on linear differential equations that Professor Bôcher gave, in which the result for the symmetric case is given without proof. Professor Bôcher states there that Weierstrass worked out this case in 1858, ten years before he introduced elementary divisors. The writer has examined the collected works of Weierstrass but he was unable to find this result.