SCHOUTEN ON RICCI-KALKÜL

Der Ricci-Kalkül. By J. A. Schouten. Berlin, Julius Springer, 1924. 312 pages.

This is Volume X of *Die Grundlehren der Mathematischen Wissenschaften,* edited by R. Courant, and is appropriately dedicated to G. Ricci, founder of the absolute differential calculus, in honor of his seventieth birthday. It is the second book on this subject published by Springer within two years. The authors (Struik and Schouten) are closely associated, and therefore the fact that the two books have many things in common is not surprising. The volume under discussion is more extended than Struik's, the additional space being devoted to a fuller discussion of non-Riemannian geometry. The methods are quite similar, but Schouten does not make use of quite so much symbolism, although Struik attributes most of his symbolism to Schouten.

These volumes give an exceedingly complete account of all the developments in differential geometry of the past five or six years, but the reviewer feels that much of the value, as reference books, is lost on account of the symbolism. The reading of many pages is often required for the full understanding of a single theorem, because it is impossible for one to keep these symbols in mind unless constantly using them.

Many of us feel that one of the best things Ricci did was to invent a consistent notation for covariant and contravariant systems. Ricci is always careful that the upper and lower indices shall have a significance. The ultra modern writers, following Weyl, have departed from this. The best example is $\Gamma_{\lambda\mu}^{\nu}$, which, according to the usual notation, should be a mixed system, covariant in λ, μ and contravariant in ν. Such is not the case, however. I, for one, fail to see why this confusion in notation should have been introduced, as it only makes the reading more laborious. One is compelled to investigate to see just what quantities are covariant or contravariant.

Geometers had been accustomed to think that Riemannian geometry was the most general geometry possible, but in the past few years has come the notion of parallel displacement, which leads to geometries quite different from that of Riemann. In the present volume, Schouten gives a very full account of these new geometries. In fact, only about fifty pages are devoted to Riemannian geometry. It does not take long to convince one that this book is entirely modern both as to material and point of view. Nearly every page has a reference to work which has appeared within six or seven years. A modern mathematical theory usually means one that is not more than fifty or