

THE CALCULUS OF VARIATIONS*

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1. *The Purpose of this Note.* It is proposed to explain briefly here the method followed in my *Fondamenti di Calcolo delle Variazioni*†, of which the first two volumes have already been published. In these the simplest problem of the calculus of variations in the plane and the corresponding isoperimetric problem are considered, for integrands which are functions only of the coordinates and the inclination of the tangent at a point of the curve along which the integrals are taken; the other problems of the calculus of variations will be discussed in later volumes.

2. *Reasons for the Development of a new Method in the Calculus of Variations.* The inconveniences which present themselves in the classical methods of the calculus of variations are well known. Here I emphasize only the fact that, aside from the difficulties inherent in the verification for a given curve of the well known sufficient conditions for an extremum, there is further a serious difficulty which sometimes arrests at the very beginning the application of the classical results. It is that the theory of differential equations offers but few methods of deciding whether or not there exists an extremal joining two points which are not conveniently near to each other. I propose on that account to find a more direct procedure for the solution of the problems of the calculus of variations, independent of the theory of differential equations. At the same time my researches have two other objectives. The first is that of obtaining general results in the domain of absolute extrema, as contrasted with the classical method which is particularly devoted to relative extrema. In the second place it will be interesting to see if the new methods

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