STUDY ON VECTORS AND INVARIANTS

Einleitung in die Theorie der Invarianten linearer Transformationen auf Grund der Vektorenrechnung. Erster Teil. By E. Study. Braunschweig, Vieweg und Sohn, 1923. 268 pp.

Like other writings of Professor Study the present work is interesting from several different angles. It is a development of the theory of invariants of ternary forms, based upon what he calls vectors; it is an exposition of his thesis that mathematics is the study* of "natural (positive, integral) numbers and everything which can be based upon them, but nothing else"; it is a running commentary and criticism of many things and many mathematicians. One needs occasionally a background of other papers of his dealing with questions of a semi-philosophical character. He particularly takes to task the investigators who have developed systems of vector analysis because they have not contented themselves with utilising the theory of invariants, as developed by Clebsch, Aronhold, etc. He says (p. 3) "Whether one develops vector analysis for the sake of its applications or as a self-contained discipline, there must always come first, as stated, a consideration of certain topics of algebra, namely invariants of certain groups of linear transformations: all expressions considered are invariants of orthogonal transformations, or invariants of groups closely related to the group of orthogonal transformations. But for more than fifty years we have had a highly developed theory of the group of all linear transformations, and for more than twenty-five years at least the fundamentals of a theory of invariants of the other groups just referred to (GES. WISS. LEIPZIG, MATH.-PHYS. 1897, p. 443 ff.). But not a glimmer of light seems to have fallen from these investigations upon the highly beloved "Vector Analysis" of today. Indeed old problems have been handled as if never treated before, and we are thus lagging far behind what has long been a guaranteed possession of the science. So far as I know, the question is never raised in such writings, as to what are all possible algebraic, and in particular, rational, invariants of these groups; and yet there can be no doubt that this is the problem, which from the very nature of things, lies at the heart of the matter." And again "As for the majority of authors it is not evident that they lived in a generation when the theory of groups was in full bloom. We even see the algorithm of H. Grassmann, which was a mark of progress in his own time, and which stands as a monument to his originality, yet which has long ago been absorbed into the more profound theory of invariants, hailed as the acme of the attainable, or even as a panacea, which certainly it is not; while on the

* *Mathematik and Physik*, p. 6.