THE GEOMETRY OF FREQUENCY FUNCTIONS*

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1. *Introduction.* The Pearson coefficient of correlation, calculated for a finite number of observations, has a geometric interpretation which is simple and almost immediate.† The same thing may be said of the corresponding expression formed for a pair of functions of a continuous variable.‡ When the distribution of the observed quantities is thought of as given by a frequency function, the geometric interpretation of the correlation coefficient is not so obvious. It is the purpose of this paper to show one form that such an interpretation may take.§ The geometric configurations are exactly the same as in the other cases mentioned; the difference is in the manner of setting up the association. This is accomplished by defining an appropriate correspondence between an arbitrary point of a plane, or of space, and an arbitrary linear combination of the variables subjected to measurement.

There will be no assumption that the distributions involved are "normal", in the sense of the Gaussian law. There will be incidental reference to frequency functions having properties that correspond to those of normal orthogonal sets of functions, as the terms are used in the theory of the development of arbitrary functions in series; but

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† Cf., e. g., D. Jackson, *The trigonometry of correlation,* AMERICAN MATHEMATICAL MONTHLY, vol. 31 (1924), pp. 275–280; also the paper cited in the next footnote.

‡ Cf., e. g., D. Jackson, *The elementary geometry of function space*; recently submitted to the AMERICAN MATHEMATICAL MONTHLY.

§ For another form, cf. James McMahon, *Hyperspherical goniometry; and its application to correlation theory for n variables,* BIOMETRIKA, vol. 15 (1923), pp. 173–208. The fundamental idea of attaching a geometric meaning to the correlation coefficient appears to be due to Pearson himself.