Suppose every set of $n-k$ congruences of the orthogonal ennuple is normal to a family V_k. Then equations (10) are satisfied when h, i and j take on any distinct values. But Ricci has shown that $\gamma_{hij} = -\gamma_{ihj}$; combining, we find $\gamma_{hij} = 0$ for h, i, j distinct, and this is a sufficient condition that all the congruences be normal. Consequently, *if every set of $n-k$ congruences ($k>1$) of an orthogonal ennuple has a family of k dimensional hypersurfaces as orthogonal trajectories, then all of the congruences are normal.*

PRINCETON UNIVERSITY

LIMITS FOR ACTUAL DOUBLE POINTS OF SPACE CURVES*

BY T. R. HOLLCROFT

1. *Introduction.* Noether† has proved that space curves of maximum genus on non-singular surfaces always exist. In the first part of this paper, such curves will be assumed to exist also when any number of actual double points are added, subject only to the fact that the genus can not be negative for a proper curve and to certain other limitations. The purpose of this paper is to ascertain and define these limitations when all of the actual double points of the space curve are cusps or when all are nodes, and to discuss the existence of space curves with any number of actual double points up to and including the maximum.

From Noether's formula for the maximum genus π_μ of a curve of order n on a non-singular surface of order μ, is obtained the theorem: The minimum number of apparent double points h_μ for a space curve of order n cut out by

---

* Presented to the Society, February 24, 1923.

† M. Noether, *Zur Grundlegung der Theorie der Algebraischen Raumkurven*, ABHANDLUNGEN DER PREUSSISCHEN AKADEMIE DER WISSENSCHAFTEN, 1882, Section 6.