NORMAL CONGRUENCES OF CURVES IN RIEMANN SPACE*

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The purpose of this note is to determine necessary and sufficient conditions that a congruence of curves in a Riemann space of n dimensions be normal to a family of k—dimensional hypersurfaces. We proceed first to find certain necessary conditions.

If the hypersurfaces, which we denote for brevity by V_k, are defined by the equations

$$f_i(x^1 x^2 \cdots x^n) = c_i, \quad (i = 1, 2, \cdots, n-k), \tag{1}$$

where the c's are constants, any one of the $n-k$ vectors, $\partial f_i / \partial x^n$, $(i = 1, 2, \cdots, n-k)$ will be orthogonal to the family. Hence, if a congruence of curves is normal to a family V_k, there are $n-k$ linearly independent congruences normal to the same family. Define these congruences by the $n-k$ systems of equations

$$\frac{dx^1}{\lambda_h|^1} = \frac{dx^2}{\lambda_h|^2} = \cdots = \frac{dx^n}{\lambda_h|^n}, \quad (h = 1, 2, 3, \cdots, n-k). \tag{2}$$

There will be no loss of generality in assuming that the congruences are mutually orthogonal; if we denote the fundamental tensor of the space by g_{ij}, that is, if the linear element is given by the positive definite form

$$ds^2 = g_{ij} dx^i dx^j, \dagger \tag{3}$$

then we may write

$$g_{rs} \lambda_h|^r \lambda_j|^s = \delta_{hj}, \tag{4}$$

* Presented to the Society, April 28, 1923.

† An index repeated, once a subscript and once a superscript, is summed from 1 to n.