ON SETS OF THREE CONSECUTIVE INTEGERS WHICH ARE QUADRATIC OR CUBIC RESIDUES OF PRIMES*

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1. *Introduction.* The problem of finding sets of two consecutive integers which are quadratic residues of a prime has been considered by a number of writers† from the point of view of finding integers x and y such that

$$x^2 \equiv y^2 + 1 \pmod{p}$$

p being a prime. I know of no references however, on the problem of finding three consecutive integers which are squares.

As to consecutive integers which are cubic residues, the congruence

$$x^3 \equiv y^3 + 1 \pmod{p}$$

has been studied,‡ but the problem of determining sets of three consecutive integers or sets in arithmetic progression which are cubic residues has apparently not been considered.

In the present note special results on such distribution of quadratic and cubic residues will be obtained.

2. *Three Consecutive Quadratic Residues.* It is known that

$$\frac{x^5 - 1}{x - 1} \equiv 0 \pmod{p}$$

has solutions prime to 5 only when the prime $p \equiv 1 \pmod{5}$.

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† See the references in Dickson, *History of the Theory of Numbers,* vol. 2, pp. 282–303 (These are included incidentally in the literature on representation of a number as the sum of four squares.)

‡ See Libri[24], Pellet[128–244], Dickson[199], Cornacchia[217], Mantel[277], Hurwitz[213], Schur[283], of Chapter 26 of vol. 2 of Dickson's *History.*