

ON THE ACCESSIBILITY OF AN ARC FROM
ITS COMPLEMENT IN SPACE OF
THREE DIMENSIONS

(Extract from a letter to R. L. Moore, dated Jan. 29, 1924)

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In a paper on continuous curves printed in the BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY, July, 1923, you have raised the question, *whether an arc lying in a three-dimensional space is accessible at each of its ends from every point which does not lie on it.*

I shall answer *positively* this question.

Let AB be an arc lying in the space S , X a point of $S-AB$, P a point of AB (not necessarily an end-point). Let H be a plane containing the points X and P .

The point-set G , composed of those points of the arc AB which lie on H , is a closed plane and bounded set. Since the arc AB passes through G , the set G does obviously satisfy a condition (which you gave with Professor Kline in a paper printed in ANNALS OF MATHEMATICS)* which is necessary and sufficient in order that it should be possible to pass an arc through it. Let T be an arc lying on H , containing G but not X . Hence T is accessible at each of its points on the plane H (see, e. g., Schoenflies). Let XP be an arc lying on H and having in common with T only the point P . Hence XP has in common with AB only the point P and thus the theorem is proved.

Obviously the proof holds true in n -dimensional space, $n \geq 3$.

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* The paper here referred to is *On the most general plane closed point-set through which it is possible to pass a simple continuous arc*, ANNALS OF MATHEMATICS, (2), vol. 20 (1919), pp. 218-223. R. L. M.