THE FREQUENCY LAW OF A FUNCTION OF ONE VARIABLE*

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1. *Introduction.* If the probalility that a variable X will take on a value not greater than x is

$$\Phi(x) = \int_{-\infty}^{x} \varphi(x)\,dx,$$

then $\Phi(x)$ is the cumulative frequency law, "Verteilung", for X; whereas the frequency law is $\Phi'(x)$, which is equal to $\varphi(x)$ at a point of continuity of $\varphi(x)$. Two closely related problems will be treated in this paper:
(1) Given the frequency law $\varphi(x)$ for a variable X, to find the frequency law $\psi(y)$ for a function Y of X;
(2) Given $\varphi(x)$ and $\psi(y)$, to find Y.

Under (1), where $Y = f(X)$ is given as a continuous increasing function, no special difficulty arises.† When, however, $f(X)$ has an infinitely multiple-valued inverse $g(Y)$, the expression naturally assignable to $\psi(y)$ will not be valid without restriction.

Two real functions $\varphi(x)$ and $f(x)$ will be introduced defined for all real values of x. In case a given $\varphi(x)$ or $f(x)$ is undefined outside a finite interval, the value zero may be assigned to it outside this interval.

As the foregoing problems belong essentially to general analysis, the two theorems to be stated will avoid the language of probability.

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† Mayr, *Wahrscheinlichkeitsfunktionen und ihre Anwendungen*, MONATSHEFTE FÜR MATHEMATIK UND PHYSIK, vol. 30 (1920), pp. 17–43. Rietz, *Frequency distributions obtained by certain transformations of normally distributed variates,* ANNALS OF MATHEMATICS, (2), vol. 23 (1922), pp. 292–300.