$$\sum_{i=1}^{m} d_i^2 = 4\sum_{i=1}^{m} \sin^2\frac{\varphi_i}{2} = 2\sum_{i=1}^{m}(1-\cos\varphi_i) = 2(m-m\bar{r}_1),$$

and $\bar{r}_1$ has the value

$$\bar{r}_1 = 1-\frac{1}{2m}\sum_{i=1}^{m} d_i^2.$$

But this reckoning does not seem to lead to any particularly simple geometric interpretation for $\bar{r}$. It may turn out, in some circumstances at least, that $\bar{r}_1$ is the more significant measure after all.

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METHODS FOR FINDING FACTORS OF LARGE INTEGERS*

BY H. S. VANDIVER

1. *Introduction.* We shall examine, in this paper, the problem of finding factors of integers beyond the range of Lehmer's factor tables, by methods shorter than that of dividing the integer by all the primes less than its square root.

Three methods will be proposed here. The first two depend on the representation of the integer as a definite quadratic form, and the third on the representation as an indefinite quadratic form. As I hope to devote another paper to the development of the last two methods, only outlines and a few examples will be given in connection with them.

The theory of quadratic forms has been applied in several different ways to the problem.†

In particular, Seelhoff‡ gave an expeditious method with the use of tables, which, however, is limited in application,

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† Dickson, *History of the Theory of Numbers*, vol. 1, pp. 361–66.

‡ AMERICAN JOURNAL, vol. 7, p. 264; vol. 8, p. 26.