SOME PROBLEMS OF CLOSURE CONNECTED WITH THE GEISER TRANSFORMATION*

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1. *Introduction.* Problems of closure may be defined as series of geometrical operations of the same type performed on a given figure with the property that the series closes after a finite number of steps and that the closure in one instance has as a consequence the closure of an infinite number of series with the same number of steps performed on the same given figure. As examples of such problems may be mentioned the well known Steiner series of circles attached to two given non-intersecting circles, the Poncelet polygons, the Steiner polygons inscribed in a cubic, etc. There are various methods of treating problems of this kind. One very effective method for a certain class of problems is by means of elliptic functions, as inaugurated by Jacobi† and Clebsch.‡

Another method, distinguished by its simplicity and directness, has been established by A. Hurwitz§ and is based on the correspondence principle in a one-parameter algebraic domain. For example, if the correspondence between the elements is (m, n) on a rational curve, there are $m + n$ coincidences. Now it is possible that in certain cases the correspondence may be such that there are more than $m + n$ coincidences. If this happens, then there are an infinite number of such coincidences and we have a

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