follows, by a theorem due to E. W. Chittenden, that* S converges relatively uniformly on the sum of all the point sets of this collection. But this sum is $E - E_0$.

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THE THEORY OF CLOSURE OF TCHEBYCHEFF POLYNOMIALS FOR AN INFINITE INTERVAL†

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1. *The Theorem of Closure.* Suppose we have a function $p(x)$, not negative in a given interval (a, b), for which all the integrals

$$\int_a^b p(x) x^n dx, \qquad (n = 0, 1, 2, \cdots)$$

exist. It is well known that we can form a normal and orthogonal system of polynomials

$$\varphi_n(x) = a_n x^n + \cdots, \quad a_n > 0, \quad (n = 0, 1, 2, \cdots),$$

uniquely determined by means of the relations

$$\int_a^b p(x) \varphi_m(x) \varphi_n(x) dx = \begin{cases} 0, & m \neq n, \\ 1, & m = n. \end{cases}$$

We call these polynomials *Tchebycheff polynomials* corresponding to the interval (a, b) with the *characteristic function* $p(x)$. The simplest example is given by Legendre polynomials, corresponding to the interval $(-1, +1)$ with $p(x) = 1$.

The most important application of Tchebycheff polynomials is their use in the development of functions into

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* E. W. Chittenden, *Relatively uniform convergence of sequences of functions,* TRANSACTIONS OF THIS SOCIETY, vol. 15 (1914), pp. 197–201. As Chittenden observes, this is an extension of a theorem given by E. H. Moore on page 87 of his *Introduction to a Form of General Analysis,* loc. cit.

† Presented to the Society, December 29, 1923.