CONCERNING RELATIVELY UNIFORM CONVERGENCE*

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According to E. H. Moore, a sequence of functions $f_1(p)$, $f_2(p), f_3(p), \cdots$, defined on a range K, is said to converge, to a function $f(p)$, relatively uniformly with respect to the scale function $s(p)$ if, for every positive number e, there exists a positive number δ_e such that if $n > \delta_e$ then, for every p which belongs to K, $|f_n(p) - f(p)| < e|s(p)|$†.

In this note I will establish the following theorem.

THEOREM. *If S is a convergent sequence of measurable functions $f_1(x), f_2(x), f_3(x), \cdots$ defined on a measurable point set E and S converges for each x belonging to E, then E contains a subset E_0 of measure zero such that the sequence S converges relatively uniformly for all values of x on the range $E - E_0$.*

PROOF. Suppose that S converges on E to the limit function $f(x)$. By a theorem due to Egoroff‡, E contains a subset E_1 of measure less than 1 such that S converges to $f(x)$ uniformly on $E - E_1$. Similarly E_1 contains a subset E_2 of measure less than $1/2$ such that S converges to $f(x)$ uniformly on $E_1 - E_2$. Continue this process thus obtaining a sequence of point sets $E_1, E_2, E_3, \cdots$ such that, for each n, (1) the measure of E_n is less than $1/n$, (2) E_{n+1} is a subset of E_n, (3) S converges uniformly on $E_n - E_{n+1}$. Let E_0 denote the set of points common to the sets $E_1, E_2, E_3, \cdots$. The set E_0 is either vacuous or of measure 0. Furthermore

$$E = E_0 + (E - E_1) + (E_1 - E_2) + \cdots.$$

Since S converges uniformly on each point set of the countable collection $E - E_1$, $E_1 - E_2$, $E_2 - E_3$, $\cdots$, it

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† See E. H. Moore, *Introduction to a Form of General Analysis*, The New Haven Mathematical Colloquium (Yale University Press, New Haven, 1910).

‡ COMPTES RENDUS, Jan. 30, 1911.