

EISENHART'S TRANSFORMATIONS OF SURFACES

Transformations of Surfaces. By Luther Pfahler Eisenhart. Published with the cooperation of the National Research Council by The Princeton University Press, 1923. IX + 379 pp.

The results of a round five score of researches in three-dimensional differential geometry, generalized to a great extent to n -space and developed largely anew to form a unified theory governed by a central idea,—this is the offering before us. The researches, with few exceptions, are the product of investigations of the last quarter of a century carried on primarily by Bianchi, Darboux, Demoulin, Eisenhart, Guichard, Jonas, Koenigs, Ribaucour, and Tzitzeica. They deal, some directly and some rather indirectly, with transformations of surfaces of a given kind into surfaces of the same kind.

Of these transformations there are two general types, transformations F , the nature of which we shall describe later, and transformations in which the two surfaces are the focal surfaces of a W congruence. Both transformations appeared first in special forms, the latter in the particular case of pseudospherical surfaces developed by Bianchi (1879) and Bäcklund (1883), the transformations F in special cases discussed by Koenigs (1891), Darboux (1899), and subsequent writers. In fact, it was not until quite recently that the general transformation F was systematically studied, by Jonas, in 1915, and by Eisenhart, in 1917. It is this theory of F , or fundamental, transformations which forms the central and unifying theme of the book.

The scope of the book is broader than the title might first suggest. The material handled is not merely abundant—besides the text there are at the end of each of the ten chapters an average of twenty-five problems serving to a large extent to summarize the results of research,—but it is also highly diversified. More than a third of the book is devoted to congruences of spheres and of circles, rolling surfaces, and surfaces applicable to a quadric, subjects which one would not at first thought relate to the theory of transformations. To unify such a diversity of material in a natural and effective fashion is not simple, and the author is to be congratulated on the masterly way in which he has succeeded. Thereby he has not only given us in many cases new methods of arriving at known results, but has brought home to us in striking fashion the breadth and power of the central theory of F transformations.

The present book is a sequel to the author's *Differential Geometry* in the sense that it refers freely to the elementary treatise for facts