A CONVERGENCE PROOF FOR SIMPLE AND MULTIPLE FOURIER SERIES*

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The purpose of this paper is to establish the convergence of both the simple and the multiple Fourier series with a second order linear homogeneous differential equation as a starting point. The method of proof for the simple series is essentially that of Birkhoff† applied to a special case. In the extension of the theory to the multiple series, the argument is similar to that used by Camp‡ in connection with a first order equation. Apart from any relation with more general theory, the proofs given here are of interest because of the elegance with which they lead to important results.

THEOREM I. *Let* $f(x)$ *be made up of a finite number of pieces in the interval* $-\pi \leqq x \leqq \pi$, *each real, continuous, and with a continuous derivative. For* $-\pi < x < \pi$ *the Fourier series for* $f(x)$ *converges to* $\frac{1}{2}[f(x-0)+f(x+0)]$. *For* $x = \pm\pi$ *it converges to* $\frac{1}{2}[f(-\pi+0)+f(\pi-0)]$.

We start with the differential equation

$$y'' + \varrho^2 y = 0, \tag{1}$$

and the boundary conditions

$$y(-\pi) = y(\pi), \qquad y'(-\pi) = y'(\pi), \tag{2}$$

where x is the independent variable and ϱ^2 is a parameter.

It is easily seen that a necessary and sufficient condition that the system (1), (2) have a solution is that ϱ be a positive or negative integer, or zero. These integral values of ϱ

* The means of approach to Fourier series which is employed in this paper was suggested to a class of graduate students by Professor R. D. Carmichael; the problem was solved completely or in part by several members of the group.

† TRANSACTIONS OF THIS SOCIETY, vol. 9 (1908), p. 390.

‡ TRANSACTIONS OF THIS SOCIETY, vol. 25 (1923), pp. 123–34.