BIRKHOFF ON RELATIVITY

Relativity and Modern Physics. By G. D. Birkhoff. Cambridge, Harvard University Press, 1923. xi + 283 pp.

Birkhoff's general plan for his book on relativity is excellent. He begins with a rapid survey of those branches of classical physics which are to be affected most seriously by the new theory. This survey shows that the old theories have been fundamentally limited by the underlying concept of a rigid body in empty space, and leads to the inquiry as to what sort of a theory can be built up by using for our physical model "a number of very small material particles in motion at comparatively great distances from each other in otherwise empty space".

For simplicity he limits attention at first to a one-dimensional case. The position of a particle B with respect to a particle A is determined by the time t_1 of emission and the time t_2 of return of a flash of light from A which is reflected by B.* Thus the events of a one-dimensional universe are seen to constitute a two-dimensional manifold. It is shown how the coordinates t_1 and t_2 may be replaced by the more conventional x, t, but the early part of the discussion is carried out in terms of the physically primordial t_1 and t_2.

By introducing certain assumptions (isometric, etc.) as to the character of space-time, attention is limited to two special cases which Birkhoff calls "aeolotropic" and "isotropic" respectively. The first leads to the classical and the second to the special relativity metric. The consequences of the two assumptions are developed far enough by the end of chapter four to give a clear general idea of the differences between the two cases. From this point on attention is restricted to the isotropic case.

Three chapters are now devoted to the development of the special relativity in a two-dimensional space-time, the main topics being the dynamics of a particle and of a system of particles and one-dimensional hydrodynamics. From this discussion there emerges that complex of ideas which Einstein has bound together in the energy-momentum tensor. Then comes a chapter on tensor analysis and Riemann geometry, followed by a chapter on gravitational theory in two dimensions.

On page 150 (there are 270 pages of text) we turn for the first time to physical problems of more than one space dimension. In two chapters

* This method of arriving physically at a coordinate system had previously been suggested by J. L. Synge, NATURE, vol. 108, p. 275 (October 27, 1921) and worked out to a certain extent for the three-dimensional case.