

that every relative integral invariant is the sum of an absolute integral invariant and the integral of a symbolic total differential, etc.

The last three chapters are devoted to recent results in the study of systems of Pfaff, a great part of which is due to Cartan. The first question considered in Chapter 6 is as to the maximum number of dimensions ρ of an integral manifold of a system of r forms of Pfaff.

There is a rich amount of material, too varied to lend itself to brief discussion, on systems of linear differential expressions, contact transformations, derived systems, the problem of Monge, second order partial differential equations, etc. The final chapter is devoted to the classification of the integral elements of a system of Pfaff and to the existence theorem.

The book affords an excellent introduction to the study of a problem which occupies a very central position in the theory of differential equations. It gives a detailed survey of the classical theory, of its connections with other domains, of its most modern developments, and of the directions in which further advances may be made.

A. DRESDEN

WATSON ON BESSEL FUNCTIONS

A Treatise on the Theory of Bessel Functions. By G. N. Watson. Cambridge, University Press, 1922. viii + 804 pages.

The purpose of this book is twofold: to develop certain applications of the fundamental processes of the theory of functions of complex variables for which Bessel functions are admirably adapted; and secondly, to compile a collection of results which shall be of value to the increasing number of mathematicians and physicists who encounter Bessel functions in the course of their researches. The author believes that the existence of such a collection is demanded by the greater abstruseness of properties of Bessel functions (especially of functions of large order) which have been required in recent years in various problems of mathematical physics.

In his exposition the author has endeavoured to accomplish two specific results: to give an account of the theory of Bessel functions which a pure mathematician would regard as fairly complete; and to include all formulas, whether general or special, which, although without theoretical interest, are likely to be required in practical applications. An attempt is made to give the latter results, as far as possible, in a form appropriate for use in the applications. These exalted aims the author seems to have achieved with a remarkable success. The