REDUCTIONS OF ENUMERATIONS IN HOMOGENEOUS FORMS*

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1. *Introduction.* By carrying out the work in detail for the form $ax^2 + by^2 + cz^2$ we shall derive a useful set of reduction formulas, and illustrate a general process which can easily be applied to the reduction of the number $N(n = f)$ of representations of the integer n in any homogeneous form f of any degree in any number of variables. This set contains implicitly the complete set of corresponding reduction formulas for $Ax^2 + By^2 + Cz^2 + \cdots + Et^2$, in any number of indeterminates $x, y, z, \ldots, t$. The formulas in no case yield by themselves a complete evaluation of $N(n = f)$ for any type of n, but in many instances they materially simplify the problem, either by making the evaluation for f depend upon that for a simpler form, or by reducing the n to be represented to a more tractable type. By means of the process developed here, combined with elliptic function expansions, I have recently obtained several new complete enumerations for special ternary and quinary quadratic forms; the results will be published in other papers.

Before proceeding to the main discussion it will be instructive to glance at what is known concerning $N(n = f)$ in the simplest case (other than f linear), viz., $f \equiv ax^2 + by^2 + \cdots$; when the degree of f exceeds 2 even partial evaluations of $N(n = f)$ are at present unknown. It seems fair to say that the simplest case of all, $N(n = x^2 + by^2)$, $b > 0$, is still far from complete; Dirichlet's well known general theorem† for the number of representations by the totality of a system of representative forms of determinant—b

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† Cf. Dickson's *History*, vol. 3, p. 19. References to the other citations of this introduction can be found by consulting the index to vol. 3, and running down the references to vol. 2.