Let us see what the effect will be when the value of z as obtained from $\varrho = 0$ is substituted in equations (1). Suppose that the substitution has been made in X and Z. It is easy to see that X_z and Z_z are equal to zero, and that to differentiate X completely with respect to x, it is necessary to differentiate with respect to x and then to use the function of a function rule, thus $X_x + X_z(\partial z/\partial x)$, and similarly for the other letters. Thus using the fact that $\varrho = 0$, we may write the equations (5) in the form

$$(16)\left\{\begin{aligned} &\left(X_p+\frac{\partial z}{\partial p}X_z\right)(Z_x+pZ_z)-\left(Z_p+\frac{\partial z}{\partial p}Z_z\right)(X_x+pX_z)=0,\\ &\left(P_p+\frac{\partial z}{\partial p}P_z\right)(X_x+pX_z)-\left(X_p+\frac{\partial z}{\partial p}X_z\right)(P_x+pP_z)=0,\\ &\left(P_p+\frac{\partial z}{\partial p}P_z\right)(Z_x+pZ_z)-\left(Z_p+\frac{\partial z}{\partial p}Z_z\right)(P_x+pP_z)=0.\end{aligned}\right.$$

It is very easy to see that these equations are now the expanded form of the determinants of the matrix (15). Hence the theorem is proved.

THE UNIVERSITY OF MISSOURI

INTEGRO-DIFFERENTIAL INVARIANTS OF ONE-PARAMETER GROUPS OF FREDHOLM TRANSFORMATIONS*

BY A. D. MICHAL

1. *Statement of the Problem.* The author† has already considered functionals of the form $f[y(\tau_0'), y'(\tau_0')]$ (depending only on a function $y(\tau)$ and its derivative $y'(\tau)$ between 0 and 1) which are invariant under an arbitrary Volterra one-parameter group of continuous transformations. The

* Presented to the Society, December 1, 1923.

† Cf. *Integro-differential expressions invariant under Volterra's group of transformations* in a forthcoming issue of the ANNALS OF MATHEMATICS. This paper will be referred to as "*I. D. I. V.*"