ON A TYPE OF PLANE UNICURSAL CURVE

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The type considered is that which meets the sides of a real triangle in points whose parameters have at most three distinct values. If we take the triangle as triangle of reference, and suitable homogeneous coordinates, we may suppose any point (x, y, z) on the curve given by the equations

$$(1)\qquad \begin{cases} x = (t-\alpha)^{p_1}(t-\beta)^{q_1}(t-\gamma)^{r_1}, \\ y = (t-\alpha)^{p_2}(t-\beta)^{q_2}(t-\gamma)^{r_2}, \\ z = (t-\alpha)^{p_3}(t-\beta)^{q_3}(t-\gamma)^{r_3}, \end{cases}$$

where

$$(2)\qquad p_1+q_1+r_1 = p_2+q_2+r_2 = p_3+q_3+r_3 = n,$$

where n is the degree of the curve.

In this, one of the quantities p_1, p_2, p_3 is zero and the other two are zero or positive integers; and so for q_1, q_2, q_3 and r_1, r_2, r_3.

We may denote the curve (1) symbolically by the array

$$(3)\qquad \begin{vmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \end{vmatrix},$$

or, as it may be written to save space,

$$(4)\qquad (p_1\ q_1\ r_1,\quad p_2\ q_2\ r_2,\quad p_3\ q_3\ r_3).$$

It is evident that (*a*) each row of (3) contains at least one zero, (*b*) the sum of the elements in each column is n, (*c*) no two columns are identical, (*d*) we get no essentially distinct curve, if we interchange two rows or interchange two columns.

The first problem that suggests itself is to find the number of essentially distinct curves of the required type; i. e., to find the number of distinct arrays (3) subject to the restrictions just mentioned. We content ourselves with giving the