

Grundzüge der Mehrdimensionalen Differentialgeometrie in direkter Darstellung. By D. J. Struik. Berlin, Springer, 1922. 198 pages.

In addition to the research papers which are published in any subject, there comes a time when one feels the need of a book which unifies the whole. The differential geometry of hyperspace has reached this point. Practically the only books on the subject are Killing's *Nichteuklidische Raumformen* and chapters in Bianchi's *Geometria Differenziale* so that the appearance of a book devoted entirely to differential geometry of n -dimensions is most welcome. This subject can well be said to have started with Riemann, followed by Christoffel, Beltrami, Lipschitz, and others who devoted a great deal of time to the study of the quadratic differential forms in n variables. These works were published between 1860 and 1880. In the latter eighties, Ricci and Bianchi began their work on n -dimensional geometry, and they and their followers have continued the work to the present time.

Ricci began his work in the absolute calculus in 1887, and he and Levi-Civita developed the subject almost to its present completeness many years ago, but it attracted but little attention. In fact it might be said that their work was practically unknown prior to 1913. Bianchi does use the covariant derivative as a notation but gives no indication of the powerful tool which Ricci made of it. However, when Einstein wrote his theory of gravitation and used the absolute calculus in its development, the subject became a live one and since that time there has been a whole army of mathematicians, scattered all over the world, working on it. The name which Ricci used was changed by Einstein to tensor calculus, and many people today are almost in total ignorance of what Ricci and Levi-Civita have done. It was, therefore, pleasant to find that Struik has dedicated his book to Ricci.

One of the outstanding features of the book is the "direct development". By introducing a sort of vector notation his work is freed from the dependence on the particular coordinate system used. The multiple algebra of the subject is developed to some extent but when one finds so many products used, it is quite a task on the memory of the reader to keep them all in mind. The notation, however, does allow one to sidestep the great mass of summation signs used by Ricci. The Clebsch-Aronhold notation is used to write forms of higher degree than the first as symbolic products of forms of the first order. Ricci's term "system" is replaced by "Affinor", and tensor is defined as a symmetric affinor. The first chapter is devoted to the multiplication of "affinors" both covariant and contravariant and mixed. In the second chapter the covariant derivative is developed and applied. The notion of geodesic parallelism introduced by Levi-Civita is made the basis of the work of this chapter. A differential is defined which bears the