

TWO RECENT BOOKS ON ALGEBRAS

Algebras and their Arithmetics. By Leonard Eugene Dickson. University of Chicago Press, 1923. ix + 241 pp.

Corpi Numerici e Algebre. By Gaetano Scorza. Messina, Giuseppe Principato, 1921. ix + 462 pp.

On first picking up Professor Dickson's book, the reader's attention is caught by the fact that the author assumes no knowledge beyond the most elementary parts of a first course in the theory of equations. As the reviewer turned from one page to another, there was a certain ease and simplicity in the style which called to mind some of those science primers of the days of our grandfathers. One felt once more the half-forgotten glamor of the rainy day long ago when one discovered several of these "primers", during one's secret investigation of a pompous old book-case on the landing of the stair.

In the first chapter, he begins with the definition of a field of numbers and the elementary notions of a linear transformation and then easily makes the transition to matrices. Having pointed out that the set of all complex numbers, $a + bi$, is an algebra over the field of all real numbers and that the set of all p -rowed square matrices with elements in any field, F , is an algebra over F , in which multiplication is usually not commutative and in which division may fail, the author defines the general algebra over any field by five postulates which differ from those used in Scorza's book in one respect. After definitions of a few elementary notions in connection with algebras, he returns to matrices and shows that the matrix algebra consisting of all two-rowed square matrices whose elements are complex numbers is carried by a linear transformation of the units into the familiar quaternions. Thus, at the very beginning, he gives an illustration of one of the most interesting and important results in the general theory of algebras, to wit, that any associative division algebra (an algebra in which division is always uniquely possible when the divisor is not zero) is equivalent, in a suitably enlarged field, to a matrix algebra.

The second chapter is a brief one giving several elementary but useful theorems about linear sets, which are the same as algebras except that they are not necessarily closed under multiplication. Although the third chapter is also rather brief, it presents, in that simple, direct manner so characteristic of most of this author's work, about a dozen theorems and corollaries grouped around the fundamental notions of subalgebra, invariant subalgebra (analogous to self-conjugate subgroup in the theory of groups), reducible algebras and simple algebras. Of