

FOUR BOOKS ON SPACE

Der Raum: Ein Beitrag zur Wissenschaftslehre. By Dr. Rudolf Carnap. Berlin, Reuter und Reichard, 1922. 87 pp.

Mathematik und Physik: Eine erkenntnistheoretische Untersuchung. By E. Study. Braunschweig, Friedrich Vieweg und Sohn, 1923. 31 pp.

Die realistische Weltansicht und die Lehre vom Raume: Zweite Auflage; Erster Teil. By E. Study. Braunschweig, Friedrich Vieweg und Sohn, 1923. x + 83 pp.

Mathematische Analyse des Raumproblems. Vorlesungen gehalten in Barcelona und Madrid. By Dr. Hermann Weyl. Berlin, Julius Springer, 1923. vii + 117 pp.

Kant described our knowledge of space and time as synthetic and a priori. By *synthetic*, he distinguished it from the analytic, more or less tautological judgments of abstract logic, while by *a priori*, he signified that it is independent of the concrete content of our senses. His account of the extensional properties of the universe was an attempt to bridge the gap between the purely abstract, ratiocinative science of geometry, and the obviously empirical nature of the space-world to which it applies.

While Kant's problem still exists, the last century has seen a tremendous change both in our notions of geometry and in our notions of the spatial world. Geometry no longer means Euclid, for since the days of Bolyai and Lobachevski we have become aware that there are other possible systems which yield no whit to the traditional geometry in the matter of logical rigor. The axioms of geometry signify no longer self-evident indubitable truths, but arbitrarily set assumptions. In short, from the mathematical standpoint, geometry is but a branch of logic, and like the rest of logic, is concerned with the consistency, the deductive sequence of its theorems, not with their truth. On the other hand, the universe is no longer treated as fitting primarily into the euclidean scheme, but into the more complicated schemes of the special or the general Einstein space-time system. For all this, the problem still remains as to how we can associate with our empirically known world of sense a mathematical structure which in at least its analysis situs properties is essentially that of Euclid, and in particular, how we can perform this association in a preliminary fashion, not merely as the final result of a long chain of careful experiments, but automatically, almost at first glance. This problem, as to the nature of our knowledge of space and time, has indeed become far more acute