

ALGEBRAS AND THEIR ARITHMETICS*

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1. *Introduction.* Beginning with Hamilton's discovery of quaternions eighty years ago, there has been a widespread interest in linear associative algebras, a subject known also under the name of hypercomplex numbers. The list of investigators in this field includes the following well known names: Hamilton, Cayley, Clifford, and Sylvester in England; Poincaré and Cartan in France; Weierstrass, Frobenius, Lipschitz, Molien, Scheffers, and Study in Germany; A. Hurwitz and Du Pasquier in Switzerland; Benjamin Peirce, C. S. Peirce, Taber, Wedderburn, Hazlett, and others in America.

Needed guides to the extensive literature on this subject are furnished by the recent book by Scorza and the two books by the writer. Many of the papers, especially the older ones, contain serious errors and obscurities. Again, a large proportion of the papers are now obsolete, since they either treat only special algebras or fail in an attempt to give a general theory, and especially since they deal only with algebras over the field of all complex numbers. But the results obtained for this very special case have since been extended to algebras over any field, and it is the latter general subject which is the really important one both for algebra and for the theory of numbers.

* A Report presented to the Society, by invitation of the program committee, at Cincinnati, December 28, 1923. This Report, the work of the author reported in it, and two other papers presented by him to the Society at the Cincinnati meetings (see this BULLETIN, page 280 of this issue) formed the basis for the award to the author of the seventy-fifth anniversary prize of the American Association for the Advancement of Science for the most notable contribution to the advancement of science presented at the meetings of that Association and its affiliated societies at Cincinnati on December 27, 1923 to January 2, 1924. (See this BULLETIN, vol. 30, Nos. 1-2, p. 90.) THE EDITORS.