# NUMBER OF CYCLES OF THE SAME ORDER IN ANY GIVEN SUBSTITUTION GROUP*

BY G. A. MILLER

1. *Introduction.* If $G$ is a transitive substitution group and if the subgroup composed of all the substitutions of $G$ which omit a given letter is of degree $n-\alpha$, then there are exactly $\alpha$ substitutions involving no letters except possibly those of $G$ which are commutative with every substitution of $G$. These $\alpha$ substitutions include the identity. If $\alpha > 1$ the remaining $\alpha-1$ substitutions may or may not appear in $G$. From this well known theorem, it follows directly that $G$ involves exactly $\alpha-1$ sets of conjugate cycles which are such that no two distinct cycles of the set have a common letter. Each of these cycles appears in $g/n$ different substitutions of $G$, where $g$ denotes the order of $G$. A necessary and sufficient condition that a transitive substitution group be regular is that no two of its sets of conjugate cycles have a common letter.

When no two conjugate cycles of $G$ have a letter in common it is evident that every pair of cycles in a set of conjugates must be commutative, but these cycles may also be commutative when $G$ is non-regular. When $G$ involves at least one set of conjugate cycles which has the property that every pair of cycles in the set is composed of commutative cycles, $G$ must be imprimitive unless all these cycles involve the same letters and are also of prime order. In this special case, $G$ is evidently always primitive. From the fact that a cycle of prime degree $p$ is transformed into each of its various powers which are incongruent to 1 (mod $p$) only by substitutions of degree $p-1$ on the letters of this cycle it results directly that such substitutions involve cycles

* Presented to the Society, December 29, 1923.