CONCERNING A SUGGESTED AND DISCARDED GENERALIZATION OF THE WEIERSTRASS FACTORIZATION THEOREM*

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1. *Introduction.* A basic theorem in the theory of analytic implicit functions, proven by Weierstrass, may for our purpose be stated as follows:

Let (1°), $f(y; x_1, \ldots, x_n)$ *be analytic at the origin and vanish there*; *and* (2°), $f(y; 0, \ldots, 0)$ *be not identically zero. Then, throughout a certain neighborhood of the origin, there holds an identity of the form*†

$$(1) \quad f(y; x_1, \ldots, x_n) = (P_0 y^m + P_1 y^{m-1} + \cdots + P_m)\, g(y; x_1, \ldots, x_n)$$

where $g(y; x_1, \ldots, x_n)$ *is analytic and does not vanish at the origin*; *and where* P_j, $j = 0, 1, \ldots, m$, *is an analytic function of* $x_1, x_2, \ldots, x_n$ *and for* $j > 0$ *vanishes when* $x_1 = x_2 = \cdots = x_n = 0$.

In his *Madison Colloquium Lectures,* Osgood called attention to the fact that the hypothesis (2°) may be omitted in the case of a function $f(y; x)$ of only two variables, without disturbing the validity of the conclusion; and suggested tentatively but without proof that the theorem in this stronger form might be true for a function $f(y; x_1 \ldots, x_n)$ of $n+1$ variables. In a later paper‡ he showed very definitely that the theorem is *not* true in general, with the omission of the hypothesis (2°). His proof of this fact consisted in the exhibition of a function of the form

$$(2) \quad f(y; x_1, x_2) \equiv x_1 - x_2 F(y)$$

which is not factorable in the form (1).

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† The theorem of Weierstrass states, in addition, that $P_0 = 1$, and that m is equal to the degree of the term of lowest degree in the series $f(y; 0, \ldots, 0)$.

‡ TRANSACTIONS OF THIS SOCIETY, vol. 17, page 4.