A CHARACTERIZATION OF SURFACES OF TRANSLATION*

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Let the non-homogeneous cartesian coordinates x, y, z of an arbitrary point on a surface be given as analytic functions of two independent variables u, v by equations of the form

$$x = U_1 + V_1, \quad y = U_2 + V_2, \quad z = U_3 + V_3,$$

where U_1, U_2, U_3 are functions of u alone and V_1, V_2, V_3 are functions of v alone. Such a surface is called a surface of translation because it can be regarded in two ways as generated by the motion of a curve which is translated so that its various points describe congruent curves. It is the purpose of this note to give another characterization of surfaces of translation, based on some notions which have arisen in the study of projective differential geometry.

Since, at each surface point P, the tangents to the curves $u =$ const. and $v =$ const. separate the two asymptotic tangents harmonically,† it follows that the parametric net is a conjugate net. Therefore the tangents to the curves $u =$ const., constructed at the points of a fixed curve $v =$ const., form a developable surface. The point where the tangent to the curve $u =$ const. through P touches the edge of regression of this developable is one of the two *ray points* of P, with respect to the parametric conjugate net. The other ray point is similarly defined on the other parametric tangent. The line joining these two ray points is called the *ray* of P. The totality of rays of all the points on the surface constitutes a congruence called the *ray congruence* of the fundamental conjugate net. And the

* Presented to the Society, April 25, 1924.

† Lie, MATHEMATISCHE ANNALEN, vol. 14 (1879), pp. 332–337.