A GENERALIZATION OF THE SYLLOGISM*

BY B. A. BERNSTEIN

The syllogism is the proposition:

If x is y and y is z, then x is z.

In the language of boolean algebra this proposition is:

If $xy' = 0$ *and* $yz' = 0$, *then* $xz' = 0$,

where the usual notations are used, with the prime indicating negation. There thus exists in boolean algebras a universal relation† R such that

$$\text{(1)} \qquad \textit{If } xRy \textit{ and } yRz, \textit{ then } xRz.$$

I propose to find the most general boolean relation R which satisfies (1), and to show the connection between this relation and the syllogism relation $xy' = 0$.

I start with the fact that any universal relation between two boolean elements x, y is given by an equation of the form

$$\text{(2)} \qquad Axy + Bxy' + Cx'y + Dx'y' = 0.$$

Let (2) be a relation R satisfying (1). Then the *discriminants* A, B, C, D must be such that from

$$\text{(i)} \qquad Axy + Bxy' + Cx'y + Dx'y' = 0,$$
$$\text{(ii)} \qquad Ayz + Byz' + Cy'z + Dy'z' = 0,$$

we may conclude

$$\text{(iii)} \qquad Axz + Bxz' + Cx'z + Dx'z' = 0.$$

That is, the discriminants A, B, C, D must be such that equation (iii) is the result of eliminating y from equations (i) and (ii). Now (i) and (ii) together are equivalent to the single equation

$$\text{(iv)} \quad (Ax + Cx' + Az + Bz')y + (Bx + Dx' + Cz + Dz')y' = 0.$$

The result of eliminating y from (i) and (ii) is, then, the result

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† That is, a relation given by a universal proposition.