WINGER ON PROJECTIVE GEOMETRY

An Introduction to Projective Geometry. By R. M. Winger. Boston, D. C. Heath, 1923. V + 443 pp.

Among the books on projective geometry in English which have appeared so far, Winger's *Introduction* is a distinct novelty. In fact it justly breaks away from the more or less traditional Cremona-Reye style of a passed period and presents, on the whole, such topics which for a more advanced study of geometry are of essential importance. For this reason, the reviewer is glad to declare from the start that Winger has written an excellent text-book.

The author himself says that "this book is intended as an introductory account for senior-college and beginning graduate students — for the prospective teacher who is seeking proper orientation of elementary mathematics, as well as the university student who lacks the preparation for an intelligent reading of the general treatises on higher geometry and the modern books on higher algebra". As mathematical preparation for a proper understanding of the book collegiate training in algebra, analytic geometry and calculus is all that is required. It is perhaps not necessary to review in detail the thirteen chapters which in order deal with essential constants; duality; the line at infinity; projective properties; double ratio; projective coordinates; the conic; collineations and involutions in one dimension; binary forms; algebraic invariants; analytic treatment of the conic; collineations in the plane; cubic involutions and the rational cubic curve; non-euclidean geometry. The book is very clearly and expressively written throughout, and the propositions are stated concisely and in simple straightforward English. Undoubtedly it will have a very refreshing effect upon the student.

The few criticisms which the reviewer wishes to make concern in some instances the method of presentation rather than the choice of contents. Thus, on page 8, the statement "to show that a condition is necessary and sufficient entails the proof of a proposition and its converse" is, of course, not obvious and needs qualification. In the definition of isotropic lines by $x \pm iy - k = 0$, it would probably be better not to introduce the new term *circular rays* for the special case when $k = 0$. The old term *ray* (*Strahl*) which has an optical meaning when qualified should be abandoned. The word *line* (straight) is sufficient for this purpose. Thus one might speak of $x \pm iy = 0$ as the principal isotropic lines. The paradoxical statements at the top of page 54 might have been omitted without harm; since from a purely geometrical standpoint they are absolutely meaningless.