$N$ and $R$. The initial form $(1_2)$ becomes one or two new quadratic forms in $N$ and $R$. We proceed similarly with a prime factor of $a/p$, etc. Finally, we obtain formulas for $x$ from $ax = \xi - by$. We conclude that all integral solutions of (8) are products of the same arbitrary integer by the numbers obtained from a finite number of sets of four expressions each quadratic in four arbitrary parameters. The explicit formulas will be discussed on another occasion.

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# ON THE REALITY OF THE ZEROS OF A λ-DETERMINANT *

BY R. G. D. RICHARDSON

Some of the best-known theorems of algebra are centered around the zeros of the polynomial in λ,

$$(1)\qquad \begin{vmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn}-\lambda \end{vmatrix} .$$

In the classical case of the determinant connected with the equations of secular variations, where the elements $a_{ij}$ are real and the determinant $|a_{ij}|$ formed from (1) by omitting the λ's is symmetric ($a_{ij} = a_{ji}$), these zeros turn out to be real. This theorem concerning the reality of the zeros has been extended † to the case where $a_{ij}$ and $a_{ji}$ are conjugate complex ($a_{ij} = \bar{a}_{ji}$). It is proposed in this note to extend it to a still more general case which has arisen in some investigations concerning pairs of bilinear forms just completed by the author. This generalization consists in allowing the coefficients of the λ's to be $n^2$ in number instead of $n$ as in (1), of allowing them to be various and complex instead of all unity, and of bordering the determinant by $m$ rows and $m$ columns. The

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† Cf. Kowalewski, *Einführung in die Determinantentheorie*, p. 130.