

where  $R$  is an arbitrary function of  $v$  and  $c$  an arbitrary constant, not zero. Whittemore's equations (1''), taken in conjunction with his relation (6), are reducible to this form.

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## THE DIFFERENTIATION OF A FUNCTION OF A FUNCTION

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The reviewer of Rothe's interesting work *Vorlesungen über Höhere Mathematik* (1921) in this BULLETIN (vol. 28, p. 468) calls attention to the author's tentative claim that the first valid proof of the formula for the derivative of a function of a function is to be found therein, and he mentions the careful treatment of the question in Pierpont's *Theory of Functions of a Real Variable* (1905).

It is perhaps worth while to notice that Dini in his *Lezioni di Analisi Infinitesimale* (autographed edition, 1877) and Genocchi-Peano in their *Calcolo Differenziale* (1884) both gave satisfactory proofs. The treatment of Genocchi-Peano is cited and reproduced by Stolz in *Grundzüge der Differential- und Integralrechnung*, Bd. I (1893).

A proof on the same lines as that of Pierpont was given by Tannery in his *Introduction à la Théorie des Fonctions d'une Variable* (1886). See also Cesàro's *Lehrbuch der Algebraischen Analysis und der Infinitesimalrechnung*, Deutsch von Kowalewski (1904), and Kowalewski's *Grundzüge der Differential- und Integralrechnung* (1909).

It is remarkable that even the most careful English writers on the calculus have missed the defect in the proof to be found in our standard works on that subject.

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