A GENERALIZATION OF A PROPERTY OF AN ACNODAL CUBIC CURVE

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1. *Introduction.* We refer to the following property.

*If in a plane cubic curve there is inscribed a real triangle ABC such that BC, CA, AB touch the curve at C, A, B, then the cubic can be projected by a real projection so as to have trigonal symmetry, i.e., it can be brought to self-coincidence by rotating it through $2\pi/3$ about a point. If, in particular, the cubic is unicursal (rational), it must be acnodal.**

The generalization suggested is that of any unicursal curve in which a triangle ABC is inscribed, so that A, B, C are each given by a single value of a parameter in terms of which the coordinates of any point of the curve are rationally expressed, while the intersections of BC, CA, AB with the curve lie respectively p at C and q at B, p at A and q at C, p at B and q at A. We shall investigate the properties of such curves.

Take the parameters of A, B, C as 0, ∞, 1.† Then choosing suitable homogeneous coordinates, we have evidently

$$(1) \qquad x : y : z = (t-1)^p : (-t)^p(t-1)^q : (-t)^q.$$

We shall find it convenient to use a quantity ϵ defined by

$$(2) \qquad \epsilon \equiv p^2 - pq + q^2.$$

Elimination of t from (1) gives

$$(3) \qquad x^{p/\epsilon}y^{q/\epsilon} + y^{p/\epsilon}z^{q/\epsilon} + z^{p/\epsilon}x^{q/\epsilon} = 0.$$

Hence the curves may be projected by a real projection so as to have trigonal symmetry, as in the case of the cubic. Points with parameters t, $1/(1-t)$, $(t-1)/t$ are those related by the symmetry. If $p = q$, p and q are factors of ϵ, and the curve is one of the "triangular-symmetric" curves discussed elsewhere.‡ We shall therefore suppose p and q unequal in

* For each non-singular or acnodal cubic, two such real triangles exist.

† See Hilton, *Plane Algebraic Curves*, Clarendon Press, p. 148. This book is referred to later as " H. P. A. C."

‡ MESSENGER OF MATHEMATICS, vol. 50, (1921), p. 171.