REPORT ON CONTINUOUS CURVES FROM THE VIEWPOINT OF ANALYSIS SITUS*

BY R. L. MOORE

1. *The Characterization of a Continuous Curve.* A *continuous curve* is the set of points represented by the pair of equations

$$x = f_1(t), \qquad y = f_2(t),$$

where $f_1(t)$ and $f_2(t)$ are continuous functions of t on the interval $I : (0 \leqq t \leqq 1)$. Thus a continuous curve is the image of the straight line interval I under a continuous † transformation which transforms each point X of the interval I into a single point $T(X)$ of the curve. In case this transformation does not throw any two distinct points of I into the same point of the curve $T(I)$, then $T(I)$ is a *simple continuous arc.* Thus a simple continuous arc is in one to one continuous correspondence ‡ with a straight line interval. If $T(0) = T(1)$, but no point of $T(I)$ except $T(0)$ is the transform, under T, of more than one point of I, then $T(I)$ is a *simple closed curve.* It easily follows that a simple closed curve is in one to one continuous correspondence with a circle.

It is clear from the above definition that every continuous

* Presented before the Southwestern Section of the Society, in somewhat different form, at the Symposium held in Lawrence, Kansas, Dec. 2, 1922.

† The point P is said to be a *limit point* of the point set M if, for every positive number ϵ, there are points of M, distinct from P, at a distance from P less than ϵ. A transformation T which throws a point set M into a point set $T(M)$ is said to be *continuous* if, in case the point P of M is a limit point of a point set N which is a subset of M, then $T(P)$ is a limit point of $T(N)$.

‡ Two point sets M and N are said to be in *one to one correspondence* if there exists a correspondence in which (*a*) to each point P of M there corresponds just one point P' of N, (*b*) no two distinct points of M correspond to the same point of N, (*c*) if the point P' of N corresponds to the point P of M, then the point P of M corresponds to the point P' of N and conversely. Such a correspondence is sometimes called a one to one *reciprocal* correspondence. For so-called one to one correspondences which are not reciprocal I prefer to use the term transformation which seems to me to be more suggestive of an operation which is thought of as taking place in one direction.