REPORT ON CURVES TRACED ON ALGEBRAIC SURFACES*

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1. *Introduction.* An extensive literature to which we propose to devote this report has grown up around the following question and the related transcendental theory. *Given an algebraic surface, what can be said in regard to the distribution of its continuous systems of algebraic curves?* † For various reasons we have chosen in place of the chronological presentation one in which function theory and analysis situs play the predominant part, and that has been made possible by two papers of Poincaré (*s*, I, II). We must however recall at the outset that the general answer to the above question had been given earlier by Severi (*u*, V, VI), his methods being largely of an algebro-geometric nature (see § 14), except for the use of a very important transcendental theorem due to Picard (*q*, II, p. 241). In favor of the methods which dominate this report, it must be stated that they alone made possible the solution of some important problems, and furthermore have notably enriched the theory.

A question similar to the above may be asked concerning algebraic varieties, but in order to remain within proper bounds, we have deemed it best to omit them altogether.

2. *Connectivity Indices.* We shall have occasion to consider throughout a basic n-dimensional manifold W_n, ‡ usually an algebraic curve ($n = 2$), or a surface ($n = 4$). A sum of closed, k-dimensional, two-sided, analytic manifolds in W_n is called a *k-cycle* of the manifold, and shall be denoted by Γ_k. If it bounds, it is a zero-cycle; the fact being expressed by a homology: $\Gamma_k \sim 0 \bmod W_n$. Homologies can be added

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† The properties of linear and continuous systems, so successfully investigated by Castelnuovo, Enriques, and Severi, have been described by them in three readily accessible reports (*q*, II, p. 485; *e*; *u*, XI). These and similar references refer to the bibliography at the end of the report.

‡ For a more extended topological discussion see *n*, II, § 1; *v*, Ch. 4.