of the form $2^n x_1$ or $x_1/2^n$. Also for these values, we see from (38) that $S(x) = j \sinh mx$, where j is plus or minus one, and must be the same for all such values of x, since (28) shows that

$$S(2x) = 2S(x)C(x). \tag{41}$$

Finally, since (28) and (37) are the addition formulas for $\sinh mx$ and $\cosh mx$, we see that these functions agree with $S(x)$ (to within a sign) and $C(x)$ for all multiples of x_1 whose fractional parts are terminating decimals in the binary scale, and hence, since all the functions concerned are continuous, at all points. Thus under I, II and III (*c*), the family is necessarily $A \sinh mx + B \cosh mx$.

7. *Conclusions.* In conclusion, we notice that since I and II alone must determine one of the three types of families discussed, we may use any characteristic property of the types in place of III. Thus, we might replace III (*a*) by the assumption "Some member of the family vanishes twice," or "Every member of the family is bounded." This last statement may be extended so as to give an alternative form of the assumption III, in terms of bounded, instead of non-vanishing functions. That is, III (*a*), (*b*) and (*c*) above may be replaced by the following postulates:

III (a'). There exist two linearly independent members of the family which are bounded.

III (b'). There exists one member of the family which is bounded, and all other bounded members of the family are linearly dependent on this one.

III (c'). No member of the family is bounded.

HARVARD UNIVERSITY

GROUPS IN WHICH THE NUMBER OF OPERATORS IN A SET OF CONJUGATES IS EQUAL TO THE ORDER OF THE COMMUTATOR SUBGROUP*

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1. *Introduction.* From the fact that the commutator quotient group is abelian, it results directly that there is no

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