years unexpected applications, for example in the theory of Riemann's Zeta-function.

VI. *Algebraic and transcendental numbers* (24 pp.): Besides the fundamental definitions and theorems (including the proof that e and π are transcendental numbers) a certain class of transcendental numbers which have been called by Maillet* *Liouville numbers* are studied. Since Maillet introduced the name and made in his book a systematic (although somewhat obscure) study of these numbers and since Perron in another work (*Die Lehre von den Kettenbrüchen*, Leipzig, 1913) gives full credit to Maillet, it is obviously an oversight that Maillet's book is not mentioned in the *Irrationalzahlen*.

The literature references are arranged for each chapter separately and seem fairly complete. However, Minkowski, *Diophantische Approximationen*, Leipzig, 1907, is not quoted. Borel, *Leçons sur la Théorie de la Croissance*, Paris, 1910, pp. 118–168, might have been mentioned in connection with chapters V, VI. It is not very satisfactory that even in the case of large books no page reference is given; a reference such as: L. Euler, *Introductio in Analysin infinitorum*, I, 1748 (a book of over 300 quarto pages, in Latin), is perhaps not easily run down.

The only American author referred to seems to be Huntington (TRANSACTIONS OF THIS SOCIETY, vol. 6 (1905).

A. J. KEMPNER

Grundlehren der Neueren Zahlentheorie. By Paul Bachmann. Volume III of Göschen's Lehrbücherei, Gruppe I, Reine Mathematik. Berlin und Leipzig, Vereinigung Wissenschaftlicher Verleger, 1921. xi + 252 pp.

The book under discussion represents the second edition of volume LIII of the well known Sammlung Schubert, G. J. Göschensche Verlagshandlung, Leipzig, 1907. No important changes have been made. New is a chronological table of the known proofs of the famous *Law of Quadratic Reciprocity:* fifty-six proofs from the year 1796 (Gauss' first proof, published 1801) to Frobenius' modification of Zeller's proof, 1914.† The mathematical basis of each proof is indicated; in thirty-two cases Gauss' lemma or a variant of Gauss' lemma is given as the foundation. A five-page alphabetic index has also been added.

Since the first edition was reviewed by J. W. Young in this BULLETIN (vol. 15 (1908–9), pp. 463-5), it is not necessary to consider in detail the mathematical contents of this excellent little book. The small corrections which were suggested in this review have been carried out.

The new edition is posthumous; it contains a five-page necrology, *Zum Gedächtnisse von Paul Bachmann*, by Robert Haussner. Bachmann's influence in stimulating interest in the theory of numbers has been so great that American readers may be interested in a few notes concerning his life and his work.

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* Maillet, *Introduction à la Théorie des Nombres Transcendants*, Paris, 1906, particularly chapters II, III.

† A less complete table is contained in Bachmann's *Niedere Zahlentheorie*, vol. I, p. 203 ff.