SHORTER NOTICES

Irrationalzahlen. By Oskar Perron. Berlin und Leipzig, Vereinigung Wissenschaftlicher Verleger, 1921. viii + 186 pp. Volume I of Göschen's Lehrbücherei, Gruppe I, Reine Mathematik.

Göschen's *Lehrbücherei* is to comprise works of textbook character, on topics chosen from the fields of mathematics, the exact sciences, and "Technik." They are intended primarily for students of universities and of *Technische Hochschulen.* Each volume is assumed to cover approximately the ground of a one semester university course.

The volume under consideration contains two parts of essentially different character. While the first half may be said to give a systematic treatment of the notion of an irrational number and its historical development, the last chapters are devoted to the interesting and not very widely known subject of the various methods of representing irrational numbers and their approximation by rational numbers and to a brief discussion of a certain class of transcendental numbers.

These last chapters, while not containing a large amount of new material, offer much that may be of interest even to professional mathematicians, particularly since the problem of approximation of irrational numbers is steadily gaining in importance and represents a difficult, but fascinating, field for research in which German and English mathematicians are making remarkable discoveries.

The book is divided into six chapters:

I. *The foundations* (32 pp.): Assuming the existence of rational numbers, the irrational numbers are defined by the Dedekind *Schnitt.* Refined questions of independence and consistency of the axioms employed are, in agreement with the character of the book, not dealt with.

II. *The notion of a limit* (28 pp.): The notions *upper and lower bound, limited, upper and lower limit* of a set of numbers are introduced. The *limit*, $\lim_{n=\infty} a_n$, is introduced only after the $\limsup_{n=\infty} a_n$, $\liminf_{n=\infty} a_n$ have been, in six pages, thoroughly established, and is defined by $\liminf a_n = \limsup a_n = \lim a_n$.—Convergent sequences and infinite series are briefly considered. The chapter ends with a four-page historical review of the various established methods of introducing irrational numbers (Cauchy, Bolzano, Weierstrass, Dedekind, Cantor, Méray, Bachmann, etc.) and a brief but clear discussion of their relations to one another.

III. *Powers and logarithms* (30 pp.): This chapter introduces powers with rational and irrational exponents, logarithms, the exponential series, etc., in the customary manner. It may be noted that the author, for formal reasons, assigns to 0^0 the value 1.

IV. *Various forms of representation of irrational numbers* (36 pp.): The following representations are explained and discussed (convergence, uniqueness, exceptional cases, etc.):

(*a*) $\gamma = \sum_{\nu=0}^{\infty} c_\nu/p^\nu$, p an arbitrary positive integer, c_ν integers, $0 \leqq c^\nu \leqq p - 1$ for $\nu \geqq 1$, $c_\nu \leqq p - 2$ for an infinite number of ν's (*systematic fractions*).