ON TRANSFORMABLE SYSTEMS AND COVARIANTS OF ALGEBRAIC FORMS*

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1. *Introduction.*—The purpose of this article is to give a rigorous demonstration of an important theorem in the theory of covariants of algebraic forms in p variables; namely, that if $(G_1, \ldots, G_h) = (0, \ldots, 0)$ is an invariantive property, the G_i being polynomials in the coefficients of the forms, there exists a set $V_1, \ldots, V_\nu$ of relative covariants in $p-1$ sets of cogredient variables, such that $(V_1, \ldots, V_\nu) = (0, \ldots, 0)$ when and only when $(G_1, \cdots, G_h) = (0, \cdots, 0)$. The corresponding theorem for invariants, i.e., for $h = 1$, is given by Bôcher, *Introduction to Higher Algebra* (p. 232). Bôcher there states that "a projective relation expressed by the identical vanishing of a covariant or contravariant is typical of what we shall usually have when a single equation is not sufficient to express the condition." This paper shows that such a projective relation can in general be characterized by the simultaneous vanishing of a number of covariants.

The special case of this theorem for binary forms is mentioned without proof by Clebsch, *Binäre Formen* (p. 91). J. P. Gram in the MATHEMATISCHE ANNALEN (vol. 7), and J. Deruyts in a book entitled *Essai d'une Théorie Générale des Formes Algébriques* (Brussels, 1891) consider the characterization by covariants of particular forms defined by the holding of identical relations among their coefficients. Both proofs are incomplete, however, and Gram's method actually leads to a false result in case the given conditions are non-homogeneous.

2. *Transformable Systems.* Consider a system of l algebraic forms in p variables

$$(1) \qquad f_i(a_{i1}, a_{i2}, \cdots a_{iq_i}; x_1, x_2, \cdots, x_p) \equiv f_i, \quad (i = 1, \cdots, l)$$

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