

SHORTER NOTICES

Vorlesungen über Zahlen- und Funktionenlehre. Erster Band, dritte Abteilung. By Alfred Pringsheim. Leipzig and Berlin, B. G. Teubner, 1921. ix + 515-976 pp.

The first part of this volume was reviewed in a previous number of this BULLETIN (vol. 25 (1919), p. 470). The second part, which appeared shortly after the first, was devoted to a detailed exposition of the theory of infinite series with real terms. The third part treats of complex numbers, infinite series with complex terms, infinite products, and continued fractions. It also contains an appendix to the whole first volume and an index. The appendix (pp. 917-969) consists of numerous references to the literature and indications of the historical development of portions of the theory, as well as further discussion of many of the topics treated in the body of the text.

The exposition exhibits Professor Pringsheim's characteristic lucidity, and the theory throughout is developed from first principles in elementary fashion. By the phrase "elementary fashion" we do not wish to connote anything in the nature of looseness of treatment or lack of rigor, for the author has been at particular pains to avoid defects of that sort. Thus the book is well adapted to meet the needs of any reader, regardless of the extent of his previous mathematical knowledge, who wishes to have a complete and logically accurate account of the arithmetic foundations of modern analysis.

In spite of the elementary treatment, the book is in no way limited to the elements of the subject and in connection with various particular topics comes close to the confines of our present-day mathematical knowledge. For example in the second chapter of Section III,* as a preparation for the treatment of the multiplication of series, the methods of Hölder and Cesàro for summing divergent series are introduced and the equivalence of these two methods, a theorem of comparatively recent date, is established. In this connection it is of interest to note that Professor Pringsheim objects to the use of the phraseology "summation of a divergent series" and introduces in place of it the expression "reduction (Reduktion) of a divergent series." He thus speaks of a certain series as being reducible (reduzibel) with a particular associated limit (zugeordneter Grenzwert). He bases his objection to the current usage on the statement that at the present time there exists no precise definition of the term "sum of a divergent series," and in his opinion an epithet so expressive and sonorous (ein so prägnant klingendes Beiwort) as summable should have a precise and definite meaning, such as the words convergent and divergent possess.

It will be recalled by those particularly interested in the study of divergent series that the objections raised by Professor Pringsheim to the present use of the word summable are quite similar to those previously expressed by Professor W. B. Ford in his book *Studies on Divergent*

* The whole first volume has been divided into four principal sections, of which Part three contains Sections III and IV.