ON THE EXISTENCE OF CURVES WITH ASSIGNED SINGULARITIES *

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The fundamental relations which connect the point and line singularities of algebraic plane curves are called Plücker's equations; they give necessary limitations upon the numbers of characteristics of the different sorts. These equations do not, however, contain in themselves any existence theorems, and after a solution in integers has been obtained, there is no guarantee that there is any curve whose characteristics are the numbers found. Erroneous views have been held on this point. The fact that a quartic is possible with three cusps suggests the existence of a unicursal curve of any order all of whose singular points are cusps; such a curve can not exist.

Various attempts have been made to show the existence of curves with assigned singularities, the most important being that of Lefschetz.† As far as simple nodes are concerned, he completed the solution, exhibiting the existence of plane curves with no singularities but simple nodes, and these in any desired number up to the theoretical maximum. He also showed that the requirement of each additional node imposed just one new condition. With regard to cusps, he was less successful. He established certain upper limits which may be attained, but the conclusions are not clean-cut, and depend upon what he calls the *Postulate of singularities*, which consists essentially in assuming that when we require a certain curve to have an additional cusp, we do not thereby impose upon it automatically more than one additional cusp.

The present paper follows closely Lefschetz's methods, but reaches a more definite and much simpler conclusion, and does so without the use of his postulate, which is proved in the course of the work. The final result is as follows.

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† *On the existence of loci with given singularities*, TRANSACTIONS OF THIS SOCIETY, vol. 14 (1913), p. 23.