

HAHN'S REELLE FUNKTIONEN

Theorie der reellen Funktionen. By Hans Hahn. Vol. I. Berlin, Julius Springer, 1921. vii + 600 pp.

A preliminary indication of the contents of a book may sometimes be conveyed by a statement as to what it does not contain. Such information is supplied by the preface to the present volume, which states that a second volume, completing the work, is to present the theory of integration and differentiation, the analytic representation of arbitrary functions, and Fourier's series. A lower inequality for the order of ideas involved is given by another statement in the preface, that although the principal facts of the general theory of sets and the theory of real numbers are summarized in an introduction for convenience of reference, no systematic development of these theories is attempted, and the difficult questions which gather around their foundations are not touched upon at all.

The prospective reader will be further enlightened by a glance at the author-index at the back of the book. There he will find thirty-eight references to Lebesgue, thirty-three to W. H. Young, and twenty-four each to Baire and Hausdorff, while, at the other end of the scale, Heine is mentioned four times, Cauchy three times, Dirichlet twice, and Riemann once.

The volume is concerned, then, with those investigations of the last few decades which have had the specific purpose of throwing the fullest and most searching light on the fundamental concepts of function and limit. In a larger sense, the working out of these concepts may be regarded as the supreme achievement of research in mathematical analysis during the past ninety or a hundred years. The various special theories, while offering more or less that is of value in their own particular conclusions, have derived an added significance from their relation to the central theme, the bringing of the mysteries of eighteenth-century and early nineteenth-century mathematics within the domain of assured knowledge and common-sense. Our files of journals doubtless contain thousands of pages which will receive scant attention from generations to come, but if this generalization of an existence theorem, or that simplification of a convergence proof, has contributed in its day to the stirring of ideas through which understanding emerges, it has done its part. From this point of view, the present work represents a somewhat comprehensive version, rather than a fragment, of what has been learned in a century of analysis. It is a summary too general in its terms, too abstract and refined, to be appreciated by limited human intelligence without a background of experience and illustration, which the reader must bring to its study; but it is an account possessing a certain symmetry and completeness of its own.

It goes without saying that a book which opens with the words "We begin with a brief survey of the simplest facts in the theory of sets," which presents the Wohlordnungssatz on page 25, and which in six hundred pages does not reach the subject of integration or differentiation, is not designed