

THE COMPLETE EXISTENTIAL THEORY OF
HURWITZ'S POSTULATES FOR ABELIAN
GROUPS AND FIELDS*

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1. *Introduction.* Hurwitz has proposed sets of postulates for abelian groups and fields.† If F' , F'' , F_n denote his sets for denumerable, continuous, and finite fields respectively, G' , G'' , G_n the corresponding sets for abelian groups, then I have proved in another paper‡ the following theorem.

THEOREM A. *Postulate-sets F' , F'' , G' , G'' , G_n ($n > 1$) are each completely independent; postulate-set F_n is completely independent§ when, and only when, n exceeds 2 and is a power of a prime.*

The object of this note is to investigate postulate-set F_n further, and to prove the following theorem, which, together with Theorem A establishes the *complete existential theory*§ of each of Hurwitz's six postulate-sets for abelian groups and fields.

THEOREM B. *For postulate-set F_n , when n exceeds 2 and is not a power of a prime, there exists no system having the character $(++++++)$, but there exist systems having all the other characters; when $n = 2$ there exist no systems having the characters $(-++++)$ and $(-+-+)$, but there exist systems having all the other characters.*

2. *Hurwitz's Postulates F_n for Finite Fields.* For finite fields Hurwitz's postulates are as follows (K , \oplus , \odot being undefined):
(A₁) If a , b , c , $a \oplus b$, $c \oplus b$, and $a \oplus (c \oplus b)$ belong to K , then $(a \oplus b) \oplus c = a \oplus (c \oplus b)$.

* Presented to the Society April 8, 1922.

† W. A. Hurwitz, *Postulate-sets for abelian groups and fields*, ANNALS OF MATHEMATICS, (2), vol. 15 (1913), p. 93.

‡ *On complete independence of Hurwitz's postulates for abelian groups and fields*, ANNALS OF MATHEMATICS, (2), vol. 24 (1922).

§ See E. H. Moore, *Introduction to a form of general analysis*, New Haven Mathematical Colloquium, Yale University Press, p. 82.