NOTE ON STEADY FLUID MOTION

BY S. D. ZELDIN

In a previous paper* I have shown how to find special invariant configurations of the projective and linearoid groups investigated by Wilczynski† in connection with steady fluid motion. It is the purpose of this note to show how the group whose general infinitesimal transformation is

$$Kf = u(x)\frac{\partial f}{\partial x} + v(x, y)\frac{\partial f}{\partial y} + w(x, y, z)\frac{\partial f}{\partial z}\ddagger$$

should be simplified in order to represent the steady motion of a fluid under the influence of forces possessing a potential.

If the external forces have a potential, then, as is known, the functions u, v, w must be such that the expression

$$Kudx + Kvdy + Kwdz$$

is a complete differential, or, what amounts to the same thing,

$$(1)\qquad \begin{aligned} \frac{\partial Ku}{\partial z} - \frac{\partial Kw}{\partial y} &= 0,\\ \frac{\partial Kw}{\partial x} - \frac{\partial Ku}{\partial z} &= 0,\\ \frac{\partial Ku}{\partial y} - \frac{\partial Kv}{\partial x} &= 0. \end{aligned}$$

Performing the operations indicated by equations (1) we get:

$$(a)\qquad \frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial x} + u\frac{\partial^2 v}{\partial x^2} = 0,$$

$$(b)\qquad \frac{\partial u}{\partial x}\cdot\frac{\partial w}{\partial x} + u\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial x\partial y} + w\frac{\partial^2 w}{\partial x\partial z} + \frac{\partial v}{\partial x}\cdot\frac{\partial w}{\partial y} + \frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial z} = 0,$$

$$(c)\qquad u\frac{\partial^2 w}{\partial x\partial y} + v\frac{\partial^2 w}{\partial y^2} + w\frac{\partial^2 w}{\partial y\partial z} + \frac{\partial v}{\partial y}\cdot\frac{\partial w}{\partial y} + \frac{\partial w}{\partial y}\cdot\frac{\partial w}{\partial z} = 0,$$

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* JOURNAL OF MATHEMATICS AND PHYSICS (Mass. Inst. of Tech.), vol. 1 (1921), p. 54.

† TRANSACTIONS OF THIS SOCIETY, vol. 1 (1900), pp. 339–352.

‡ This class of groups has been investigated by Sophus Lie in connection with two-point invariants. See Lie-Engel, *Theorie der Transformationsgruppen*, vol. 3, Abtheilung 5.